Pricing Weather Derivatives for Extreme Events

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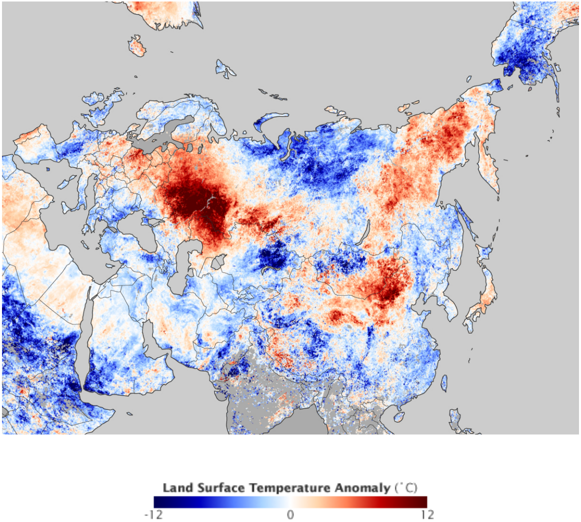
Motivating Example: Russian Heatwave, July 2010



Source: NASA Earth Observatory

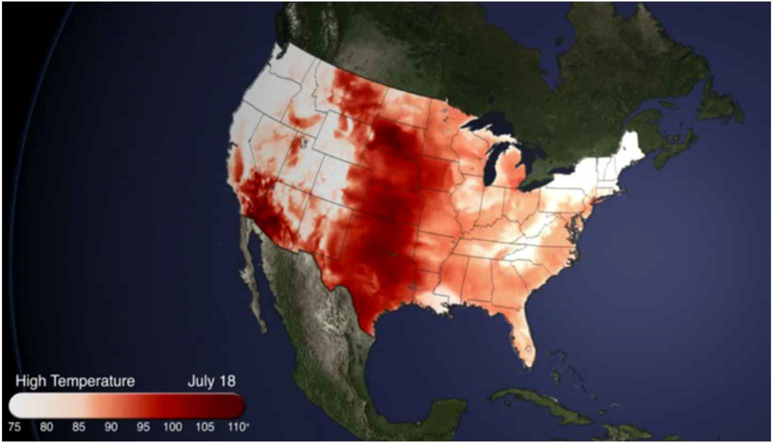
Motivating Example: Russian Heatwave, July 2010

Motivating Example: Russian Heatwave, July 2010

Source: European Space Agency, July 29 2010

Motivating Example: Russian Heatwave, July 2010

Motivating Example: US Heatwave, July 2011



Weather Derivatives

1. Weather index
2. Well-defined time period
3. Weather station used for reporting
4. Payment $L(m; s, t)$, where m is weather value, and s, t are strike and limit values

Example: Loss is $1,000 per degree if maximum daily temperature in Phoenix, AZ exceeds 116 in the month of August

Three steps to procedure:

1. Model extremes of weather process
2. Monte Carlo weather simulations $\rightarrow$ Monte Carlo simulated payments
3. Estimate risk-loaded premium as $\hat{P} = \hat{E}(L) + \lambda \cdot \widehat{\text{var}}(L)$

Generalize Extreme Value distribution

- Let $Y_1, ..., Y_n$ be i.i.d. from F
- Define $M_n = \max(Y_1, ..., Y_n)$

If there exist sequences of constants $a_n > 0$ and b_n such that

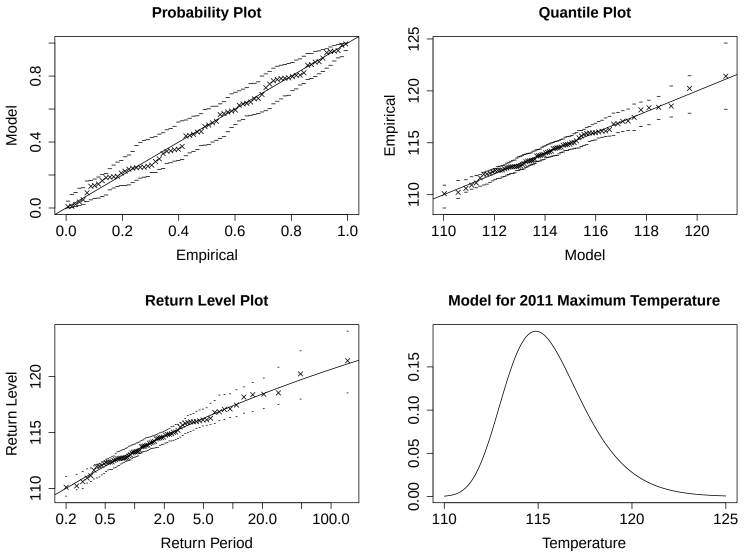
$$\lim_{n \to \infty} F\left(\frac{M_n - b_n}{a_n} \leq z\right) \to G(z)$$

for some non-degenerate distribution function G, then G is a member of the *Generalized Extreme Value* (GEV) family, and

$$G(z) = \exp\left[-\left(1 + \xi \frac{z - \mu}{\sigma}\right)_+^{-1/\xi}\right]$$

Here $a_+ = \max(a, 0)$, and μ, σ, and ξ are the location, scale, and shape parameters, respectively

Example: Maximum Summer Temperature in Phoenix, AZ



Example: Maximum Summer Temperature in Phoenix, AZ

Recall premium is $\hat{P} = \hat{E}(L) + \lambda \cdot \widehat{\text{var}}(L)$

Estimate moments using Monte Carlo simulation

$$\frac{1}{I}\sum_{i=1}^{I} L(m_i)^d \to E(L(M)^d) = \int L(m)^d g(m)\, dm \qquad \text{(almost surely)}$$

Example: Derivative pays $L = \max(1,000(M - s), 0)$ for maximum temperature M in Phoenix, AZ

Threshold s	114	116	118	120	122	124
$\hat{E}(L)$	1,882.13	732.20	224.57	56.39	11.59	1.87
$\hat{E}(L^2) \cdot 10^{-3}$	7,336.56	2,369.34	627.82	137.98	24.45	3.26

For $s = 116$, $\hat{P} = 732.30 + 1,833,223.2 \cdot \lambda$

Extension to Spatial Extremes: Max-stable Processes

- Let $Y(x)$ be a non-negative stationary process on $X \subseteq \mathbb{R}^p$ such that $E(Y(x)) = 1$ at each x.
- Let Π be a Poisson process on $\mathbb{R}_+$ with intensity $s^{-2}ds$.

If $Y_i(x)$ are independent replicates of $Y(x)$, then

$$Z(x) = \max s_i Y_i(x), \qquad x \in X$$

is a stationary max-stable process with GEV margins.

"Rainfall-storms" interpretation: think of $Y_i(x)$ as the shape of the i^{th} storm, and s_i as the intensity.

Realization of a Max-stable Process

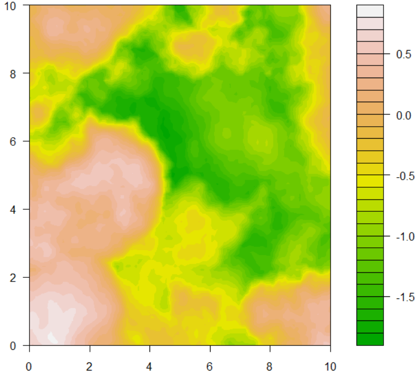


Figure: Extremal Gaussian process with Whittle-Matérn correlation with nugget=1, range=3, and smooth=1

Composite Likelihood

The joint likelihood function cannot be written in closed form for more than 2 locations. Substitute composite log-likelihood:

$$\mathcal{L}_C = \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{i=j+1}^{I} \log(f(x_{i,n}, x_{j,n}; \theta))$$

- Maximizing numerically yields $\hat{\theta}_{MCLE} = \text{argmax}_{\theta} \mathcal{L}_C$
- $\hat{\theta}_{MCLE} \sim N(\theta, I(\theta)^{-1})$, where $I(\theta) = H(\theta)J^{-1}(\theta)H(\theta)$, $H(\theta) = E(-H(\mathcal{L}_C))$, $J(\theta) = \text{var}(D(\mathcal{L}_C))$

Example: Pricing a Portfolio of Weather Derivatives

For a single derivative, risk load varies with variance

$$R(L) = \lambda \cdot \mathrm{var}(L)$$

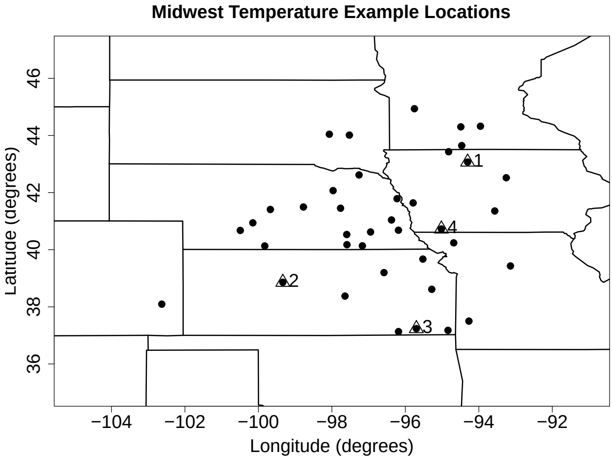
For a K^{th} derivative, risk load varies with *marginal variance*

$$R(L_K) = \lambda \left(\mathrm{var}(L_K) + 2 \sum_{j=1}^{K-1} a_{j,K} \cdot \mathrm{cov}(L_j, L_K) \right)$$

where $a_{j,K}$ is chosen to fairly split covariance; one possibility is

$$a_{j,K} = \frac{E(L_K)}{E(L_j) + E(L_K)}$$

Example: Midwest Temperature Portfolio



Example: Midwest Temperature Portfolio

Event	L_1	L_2	L_3	$\sum_{j=1}^{3} L_j$	L_4	$\sum_{j=1}^{4} L_j$
1	0	0	0	0	0	0
2	0	757.76	0	757.76	0	757.76
3	0	0	0	0	0	0
4	1,000	964.02	0	1,964.02	444.94	2,408.96
...	...	...	...	...	...	...
100,000	0	0	0	0	0	0
Mean	221.75	96.751	11.892	330.393	55.271	385.664
Variance $(\cdot 10^{-3})$	172.58	99.89	6.35	381.38	46.95	561.98
$\text{Cov}(L_j, L_4)(\cdot 10^{-3})$	28.46	29.93	8.43	66.82		
$\hat{a}_{j,4}$	0.1995	0.3636	0.8229			

$$\begin{aligned} \hat{P}(L_4) &= \hat{E}(L_4) + \lambda \left(\widehat{\text{var}}(L_4) + 2 \sum_{j=1}^{3} \hat{a}_{j,4} \cdot \widehat{\text{cov}}(L_j, L_4) \right) \\ &= 55.271 + 93,944.73 \cdot \lambda \end{aligned}$$

Conclusion

- Model targets extremes and incorporates spatial dependence
- Uses Monte Carlo simulations to obtain moments of payments $L(m)$
- Computes risk-loaded premiums

- Future research: Bayesian model fitting through approximate Bayesian computing, which incorporates parameter uncertainty into risk-loaded premiums

Thanks.