

## Sensitivity Analysis with Chi-square divergences

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An approach to sensitivity analysis for quantitative risk models, such as those used in solvency calculations is introduced. The aim of the sensitivity analysis method is to identify the most influential – and hence most important – inputs of a model. The approach follows two steps:

- 1) A change of measure is used as a tool to re-weight different scenarios (states of the world). The Radon-Nikodym density is derived by minimizing the chi-square divergence under a constraint (stress) on the expectation of a chosen random variable (model input or output).
- 2) The change of measure is used to produce a stress on the distribution of any variable of interest. The distortion in the empirical distribution of the model inputs and output are quantified.

We give an explicit solution to the divergence minimization problem, which yields a Radon-Nikodym density that is a piecewise linear function of the random variable whose expectation is being stressed. The sensitivity analysis method is illustrated through a numerical example of a simple insurance portfolio. We follow two distinct approaches: in the first we stress the expectation of the output and monitor the distribution of the inputs; in the second, we stress the expectation of all inputs in turn, and evaluate the change in (risk measures of) the output. Specific sensitivity measures are also introduced to evaluate the sensitivities of the input factors such that a high sensitivity value would be tantamount to a substantial change in the distribution of an input factor. Further, a simulation study is used to quantify the sampling error in the calculation of the sensitivity measures, showing that the Chi-square divergence performs better than Kullback-Leibler divergence in that respect.

The approach taken in this paper gives an applicable alternative to common sensitivity analysis methods used in the insurance industry: as it provides a consistent manner of stressing risk factors and only requires a single set of simulated input/output scenarios, resulting in easy implementation.